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A Fama-MacBeth Analysis of CAPM, the Fama-French Three-Factor Model and the Fama-French Five-Factor Model on 25 Stock Portfolios

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Abstract:

This paper tests the Capital Asset Pricing Model (CAPM), the Fama-French three-factor model, and the Fama-French five-factor model using 25 size and book-to-market sorted portfolios over the period July 1963 to November 2013 to examine the comparative performance of these well-established asset pricing models in explaining cross-sectional variation of returns. I employ the Fama-MacBeth (1973) methodology to address cross-sectional correlation issues inherent in asset pricing tests. Empirical investigation reveals that market beta alone fails to capture the variation in average returns. In contrast, the Fama-French three-factor model, which incorporates size (SMB) and value (HML) factors alongside market risk, substantially improves explanatory power. Moreover, incorporating two more additional factors, profitability (RMW) and investment (CMA), - known as the Fama-French five-factor model - enhances explanatory power but the improvement is marginal. However, both models have lower root mean square alpha and higher average R-squared compared to CAPM. Fama-MacBeth approach of estimating coefficients confirms that book-to-market equity, profitability, and investment factors exhibit strong explanatory power for stock returns while market beta, and size factors do not sufficiently capture the heterogeneity in stock returns. The diminishing power of the size effect has significant implications for portfolio management strategies based on the size effect. This study corroborates the superiority of the five-factor model over three-factor model and single-factor CAPM in describing the cross-sectional pattern of stock returns for this extended sample period.

Keywords: Asset pricing, CAPM, Fama-French three-factor model, Fama-French five-factor model, Fama-MacBeth, Cross-sectional returns, Size effect.

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1.0 Introduction

The attempt to comprehend and explain asset returns has been a primary interest in financial economics for decades. Asset pricing models serve as theoretical frameworks that seek to discover the systematic variables driving cross-sectional variances in expected returns across different assets. These models are crucial to modern finance, providing insights into risk measurement, portfolio construction, and investment decision-making processes.

Among the asset pricing models, the Capital Asset Pricing Model (CAPM), established by Sharpe (1964), Lintner (1965), and Mossin (1966), represents the cornerstone of modern asset pricing theory. CAPM maintains that the expected return of an asset is linearly proportional to its systematic risk, measured by beta, which encapsulates the asset's sensitivity to market movements. Despite its theoretical elegance and widespread adoption in practice, extensive empirical testing has revealed significant limitations in CAPM's ability to fully explain the cross-section of stock returns (Roll, 1977; Banz, 1981; Basu, 1983; Fama and French, 1992, 1993, 1996, 2004; Jagannathan and Wang, 1996).

The shortcomings of CAPM led to the development of multifactor models, most notably the Fama-French three-factor model introduced by Fama and French (1993). This model enhances CAPM by including two additional risk factors: size (small minus big, SMB) and value (high minus low book-to-market equity, HML). The inclusion of these criteria was inspired by significant empirical evidence suggesting that small-cap stocks and value companies tend to outperform their large-cap and growth equivalents, respectively, even after correcting for market risk (Banz, 1981; Reinganum, 1981; Basu, 1983; Rosenberg et al., 1985; De Bondt and Thaler, 1985; Chan et al., 1991; Fama and French, 1992, 1993).

Asset pricing models aim to explain the cross-section of asset return by uncovering factors that hold information about return. Empirical testing of these models provides evidence that factors in specific model describe the cross-section of expected returns consistently. The empirical validation of asset pricing models involves distinct methodological obstacles, particularly the issue of cross-sectional correlation across error terms. Although numerous ways to test asset pricing models are available, Fama and MacBeth (1973) methodology is extensively employed since it tackles the cross-sectional correlation problem. The Fama-MacBeth (1973) methodology addresses this worry through a two-step procedure that first estimates factor loadings using time-series regressions and subsequently investigates the cross-sectional link between average returns and predicted betas. This approach has become the standard way for testing asset pricing models and evaluating their explanatory power (Petersen, 2009; Cochrane, 2005).

This study provides an empirical evaluation of prominent asset pricing models' ability to explain the cross-sectional variation in stock returns. Employing the Fama-MacBeth (1973) methodology on 25 portfolios formed on size and book-to-market from July 1963 to November 2013, the research sequentially builds its inquiry. It begins by testing the efficacy of the traditional Capital Asset Pricing Model (CAPM) to determine if market beta alone sufficiently explains average returns. The analysis then assesses the incremental explanatory power of the Fama-French three-factor model, examining whether the inclusion of size (SMB) and value (HML) factors significantly reduces the pricing errors inherent in the CAPM. Subsequently, the investigation extends to the Fama-French five-factor model to ascertain if the addition of profitability (RMW) and investment (CMA) factors provides a more comprehensive explanation of return variations. The study culminates in a comparative performance analysis of these competing models, using metrics such as average R^2 and root mean squared alpha to ultimately determine which of the proposed risk factors command a statistically significant premium in the cross-section of expected returns.

By evaluating the CAPM, the Fama-French three-factor model, and the Fama-French five-factor model on 25 Fama-French stock portfolios formed on size and book-to-market equity (BE/ME), this study provides corroboration that Fama-French five-factor model beats both Fama-French three-factor model and CAPM in explaining cross-section of average return on 25 Fama-French portfolios during the July 1963 – November 2013 timeframe. Besides, book-to-market equity, profitability, and investment factors capture considerable heterogeneity in stock returns while market beta, and size factors, do not sufficiently describe the cross-sectional variation in stock returns. Failure of well-known size factor to explain the heterogeneity in stock returns is in line with the recent findings that size effect has diminished or disappeared since the 1980s (Schwert, 2003; Ahn, Min and Yoon, 2019).

This work contributes to the large literature on asset pricing by providing a detailed comparison of CAPM, the Fama-French three-factor model, and the Fama-French five-factor model using the Fama-MacBeth approach. Besides, this investigation focuses on 25 portfolios constructed on size and book-to-market equity characteristics, investigating monthly returns on 25 Portfolios formed on size and book-to-market ranging from July 1963 to November 2013. This extended time period enables robust testing across diverse market circumstances and economic cycles, offering vital insights into the relative performance of these competing models. The value of this research resides in its ability to inform both academic understanding and practical investing strategies. For academics, the findings contribute to the current discussion concerning the right specification of asset pricing models and the sources of risk premiums in financial markets. For practitioners, understanding which factors regularly explain return differences is critical for risk management, portfolio optimization, and performance evaluation.

This paper proceeds as follows. Section 2 describes background of three asset pricing models tested in the study. Section 3 enumerates the details of data and methods applied. Section 4 offers empirical evidence on performance of each of the asset pricing models and draw comparative analysis. Section 5 concludes the paper.

2.0 Literature Review

2.1 Development and Testing of CAPM

The Capital Asset Pricing Model emerged from the portfolio theory framework developed by Markowitz (1952) and represents one of the most influential theories in financial economics. The model's basic thesis is that in equilibrium, the expected return of any asset should be directly proportional to its systematic risk, as measured by beta. Early empirical tests by Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973) produced inconsistent evidence about CAPM's validity.

The pioneering work of Fama and MacBeth (1973) created the methodological groundwork for testing asset pricing models by resolving the cross-sectional correlation problem inherent in traditional regression methodologies. Their two-step technique involves first estimating factor loadings by time-series regressions and then investigating cross-sectional connections between average returns and estimated factor loadings. This methodology has established the gold standard for asset pricing testing due to its ability to deliver consistent estimates and valid statistical inference.

Despite initial support, additional study showed substantial abnormalities that undermined CAPM's empirical validity. Basu (1977) reported the earnings-price effect, whereas Banz (1981) found the size effect, showing that small-capitalization equities generated better risk-adjusted returns than projected by CAPM. Stattman (1980) and Rosenberg, Reid, and Lanstein (1985) offered early evidence of the book-to-market effect, revealing that firms with high book-to-market ratios experienced greater performance relative to their CAPM-predicted returns. Fama and French (2004) disproved the CAPM model's prediction of a linear link between beta and expected returns by demonstrating that CAPM is unable to account for the volatility in stock returns. Because of this, a lot of scholars contend that several risk factors are required to properly capture the cross-section of expected returns and that single-factor models are insufficient for explaining asset returns.

2.2 The Fama-French Three-Factor Model

The accumulation of empirical abnormalities led Fama and French (1992) to perform a comprehensive investigation of the cross-section of stock returns. Their revolutionary work proved that size and book-to-market equity simultaneously captured the cross-sectional variation in average stock returns, while beta had limited explanatory power when both variables were included. This conclusion marked a substantial challenge to CAPM's theoretical foundation and operational applicability.

Building on these empirical findings, Fama and French (1993) developed their three-factor model, which augments CAPM with size and book-to-market components. The model specification includes the market factor (similar to CAPM), a size factor (SMB - Small Minus Big) that captures the return difference between small and large-cap stocks, and a value factor (HML - High Minus Low) that reflects the return differential between high and low book-to-market stocks.

The theoretical foundation for the Fama-French factors has been a source of ongoing discussion. Fama and French (1993, 1996) suggest that size and book-to-market capture systemic risk indicators associated to financial hardship and future earnings expectations. Alternative explanations include behavioral biases (Lakonishok, Shleifer, and Vishny, 1994) and data mining problems (Lo and MacKinlay, 1990). Despite these disputes, the empirical resilience of the three-factor model has been established across multiple time periods, market segments, and foreign markets.

2.3 The Fama-French Five-Factor Model

Fama and French (2015) expanded their initial framework - Fama-French Three-Factor Model with additional factors. Novy-Marx (2013) demonstrated a robust profitability impact, demonstrating that firms with higher operating profitability tend to generate superior risk-adjusted returns which can not be explained by the three-factor model. Simultaneously, Titman, Wei, and Xie (2004) and Cooper, Gulen, and Schill (2008) offered evidence of an investment effect, demonstrating that firms with conservative investment policies are likely to outperform those with aggressive asset growth, which has not been accounted in existing factor models. Fama and French (2015) introduced their five-factor model, which expands the three-factor framework with profitability and investment factors. The model specification retains the market, size, and value factors while adding a profitability factor (RMW - Robust Minus Weak) that captures the return spread between firms with high and low operating profitability, and an investment factor (CMA - Conservative Minus Aggressive) that reflects the return differential between firms with conservative and aggressive investment strategies. This expanded framework aims to provide a more comprehensive overview of the cross-sectional variation in stock returns.

The theoretical justification for these additional factors stems from valuation theory, where Fama and French (2015) argue that expected returns should be proportional to propensity of profitability and investment through their effects on discount rates and expected cash flows. Higher expected profitability implies higher expected returns, while higher investment, maintaining profitability constant, implies lower expected returns. However, the empirical performance of the five-factor model has revealed shortcomings. Fama and French (2015) assert that the value factor (HML) becomes redundant in the presence of the other four factors, particularly in describing average returns.

2.4 Comparative Studies and Extensions

Numerous research papers have examined the explanatory power of CAPM and the Fama-French three-factor model. Davis, Fama, and French (2000) extended the analysis to international markets, finding that the three-factor model consistently outperforms CAPM across developed markets. Similarly, Griffin (2002) proved the higher performance of the three-factor model in explaining domestic portfolio returns compared to a single-factor overseas model.

The resilience of the Fama-French model has been examined across several portfolio construction approaches. Brennan, Chordia, and Subrahmanyam (1998) studied the model’s performance on portfolios constructed on diverse characteristics, while Daniel and Titman (1997) investigated whether the components reflect risk or represent return premiums linked with business features. Their findings show that the book-to-market effect may be more directly tied to firm characteristics than to systematic risk variables.

Recent versions of the three-factor model have integrated additional elements to address lingering anomalies. Carhart (1997) added a momentum element to create a four-factor model, while Fama and French (2015) introduced profitability and investment factors in their five-factor model. Pastor and Stambaugh (2003) introduced liquidity as an additional risk factor, and more recently, behavioral components have been proposed to capture investor attitude and market anomalies. Harvey, Liu, and Zhu (2016) warned that the growth of factors in asset pricing models can be the result of data mining rather than real risk factors, and they advise that slight increases in explanatory power should be carefully considered.

2.5 Methodological Considerations

The choice of test technique considerably determines the findings drawn about asset pricing models' performance. While the Fama-MacBeth methodology remains extensively utilized, newer methodologies such as the generalized method of moments (GMM) have gained prominence. Jagannathan and Wang (1996) revealed that the choice of test methodology can alter conclusions concerning CAPM's validity, particularly when considering time-varying betas and human capital considerations.

Portfolio building approaches also play a key part in empirical testing. The typical strategy involves constructing portfolios based on size and book-to-market characteristics, as suggested by Fama and French (1993). However, alternate formation methods, such as those based on industry classifications or other business characteristics, can provide different results concerning model performance.

The statistical power of asset pricing tests has been a worry addressed by various scholars. Lewellen, Nagel, and Shanken (2010) suggested that many asset price tests lack adequate capacity to distinguish between competing models, particularly when utilizing portfolios as test assets. This concern highlights the importance of careful test design and the interpretation of empirical results.

2.6 Implications for Portfolio Management and Risk Assessment

The practical ramifications of asset pricing model performance extend beyond academic interest to real-world portfolio management and risk assessment applications. The superior explanatory power of the Fama-French five-factor model and three-factor model has led to its widespread adoption in performance evaluation, cost of capital estimation, and risk management practices.

Performance evaluation using the five-factor model and three-factor model allows for more accurate risk adjustment by controlling for respective factor effects. This revision is particularly relevant for measuring the performance of active portfolio managers and mutual funds, since standard CAPM-based measurements may produce misleading estimates of managerial skill.

The evolution from single-factor to multifactor models reflects the ongoing refinement of our understanding of risk and return relationships in financial markets. While the argument over the right specification of asset pricing models continues, the empirical data consistently supports the inclusion of size and value parameters in explaining the cross-section of stock returns. This study contributes to this literature by providing updated evidence on the relative performance of CAPM, the Fama-French three-factor model, and the Fama-French three-factor model using a comprehensive dataset and rigorous methodology.

3. Data and Measures

3.1 Data

I use Kenneth R. French - Data Library[2] to construct sample of this study. Monthly returns on 25 Portfolios formed on size and book-to-market are from the Kenneth R. French site. Monthly market returns, risk free rate, average returns on size, value, profitability, and investment factors come from the same site. Though, data on market returns, risk free rate, average returns on size, value factors are available from 1926, I follow Fama and French (1993) to sample data from 1963 to 2013. Since, data on profitability and investment factor are available from July 1963, I use sample period July 1963 – November 2013 for testing CAPM and Fama-French three factor model and July 1963 – November 2013 for testing Fama-French five factor model. Final sample contains 605 monthly observations for testing CAPM, Fama-French Three factor model and Fama-French Five Factor model, respectively.

3.2 Empirical Specification

Portfolio returns include both current income and capital gain. Fama-MacBeth (1973) methodology has been applied to test each asset pricing model. Firstly, factor loadings on each model have been estimated by time-series regressions on rolling basis. Rolling window for time-series regression is 60 months. Subsequently, cross-sectional regressions have been carried out to estimate the factor risk premium and alpha. Metrics such as R^2 and Root Mean Square (RMS) alpha to assess model fit.

CAPM asserts that expected return on assets is proportional to market beta. It projects linear relationship between risk premium $(R - Rf)$ and beta (β). If this projection really works, pricing error (α) must be indifferent from 0.

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[2] https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

$$R_{k,t} - Rf = \alpha_k + \beta_k[Rm_t - Rf] + \varepsilon_{k,t}, \quad (1)$$

Fama-French three-factor model postulates that overall market $(Rm - Rf)$, book-to-market equity (HML), and size (SMB) factors produce common variation in average stock returns. If these three factors adequately capture common variation in stock returns, the intercept (α) in the time-series regression of excess return on these three factors should be indistinguishable from 0.

$$R_{k,t} - Rf = \alpha_k + \beta_k(Rm_t - Rf) + h_k HML_t + s_k SMB_t + \varepsilon_{k,t}, \quad (2)$$

Fama-French five-factor model proposed two additional factors – operating profitability (RMW) and investment (CMA) factors in addition to the overall market $(Rm - Rf)$, book-to-market equity (HML), and size (SMB) factors with argument that these five factors together can capture most of the common variation in average stock returns. If these five factors can explain most of the common variation in stock returns, the intercept (α) in the time-series regression of excess return on these five factors is expected to be not distinguishable from 0.

$$R_{k,t} - Rf = \alpha_k + \beta_k(Rm_t - Rf) + h_k HML_t + s_k SMB_t + r_k RMW_t + c_k CMA_t + \varepsilon_{k,t}, \quad (3)$$

4. Empirical Analysis

4.1 CAPM

Table 1 presents monthly average excess returns on 25 Fama-French portfolios. Average excess returns on these portfolios range from 0.27% to 1.13% per month. Moreover, average returns tend to be higher from the big- to small-size portfolios except for the lowest-BE/ME quintile. Similarly, average returns on the highest-BE/ME portfolios are consistently higher relative to the lowest-BE/ME portfolios. The difference between the average returns on the highest- and lowest-BE/ME portfolios varies from 0.21% to 0.86% per month. If CAPM really works on 25 portfolios, these obvious patterns of systematic variation in returns are to be captured by market betas.

Table 1. Summary Statistics and CAPM Regressions for Simple Monthly Percent Excess Returns on 25 Fama-French Portfolios Formed on Size and BE/ME: July 1963 to November 2013, 605 months.

$$R_{k,t} - Rf = \alpha_k + \beta_k[Rm_t - Rf] + \varepsilon_{k,t}$$

	Book-to-market equity *(BE /ME)* quintiles									
Size	Low	2	3	4	High	Low	2	3	4	High
quintiles	$\overline{R(t) - Rf(t)}$					β				
Small	0.27	0.80	0.80	1.02	1.13	1.43	1.23	1.11	1.02	1.07
2	0.46	0.74	0.89	0.94	1.02	1.40	1.18	1.05	1.01	1.12
3	0.48	0.80	0.75	0.89	1.06	1.33	1.12	1.00	0.96	1.05
4	0.59	0.59	0.68	0.87	0.84	1.23	1.08	1.01	0.96	1.08
Big	0.45	0.51	0.52	0.48	0.66	0.98	0.94	0.84	0.88	0.95
	α					$t(\alpha)$				
Small	-0.44	0.19	0.26	0.51	0.61	-2.23	1.11	1.82	3.55	3.96
2	-0.22	0.17	0.37	0.44	0.47	-1.51	1.40	3.28	3.96	3.33
3	-0.18	0.25	0.26	0.42	0.54	-1.43	2.63	2.82	4.23	4.22
4	-0.01	0.06	0.18	0.40	0.31	-0.10	0.76	2.04	4.28	2.47
Big	-0.04	0.04	0.10	0.04	0.19	-0.59	0.63	1.17	0.42	1.39

Note: Rm is the value-weighted monthly return (%) of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ. Rf is the one-month Treasury bill rate, observed at the beginning of the month. Dependent variable: Excess returns on 25 portfolios of stocks formed on size and book-to-market equity. β is slope for market factor and α is intercept from time-series regression for 25 portfolios. Rolling window for time-series regression is 60 months.

Table 1 reports estimate of the CAPM time-series regression. Although market betas are higher for small-stocks, betas are surprisingly lower for high-BE/ME stocks. It indicates that variation due to size is somewhat explained by betas, but it does not capture variation in returns caused by BE/ME. Alphas for the smallest-size portfolios surpass those for the biggest by 0.15% to 0.47% per month except for the lowest-BE/ME quintile. Moreover, significant alphas for 13 portfolios suggest that CAPM leaves a large unexplained return on 25 portfolios. However, joint test of alphas would offer reliable statistics to test significance of pricing error.

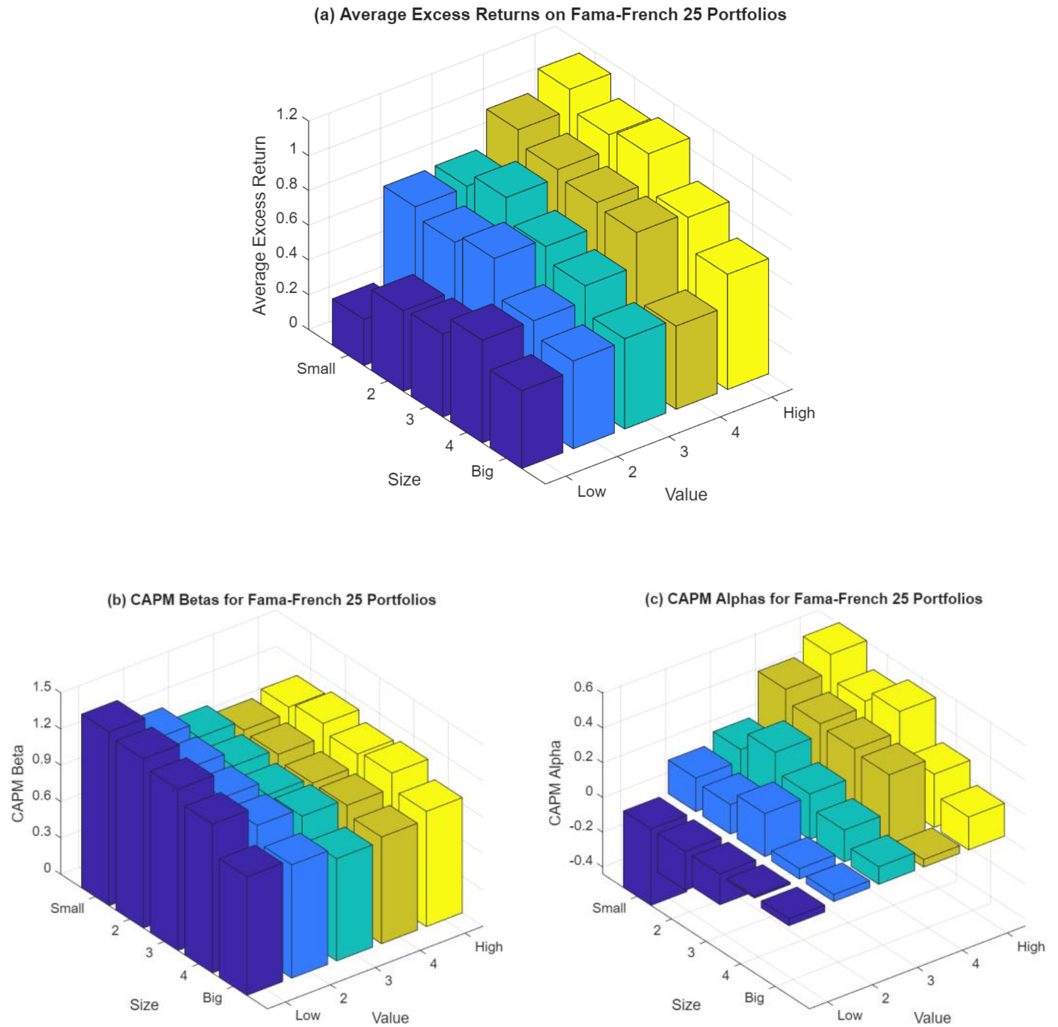

Figure 1. Top panel exhibits average excess returns on Fama-French 25 portfolios with average excess return in z-axis. Left part and right part of bottom panel present betas and alphas, respectively which are generated from time series regression of excess return for each of Fama-French 25 portfolios on excess market return. Z-axis in left figure and right figure of bottom panel indicates the CAPM beta and CAPM alpha, respectively. X-axis and Y-axis of all plots show sorting of portfolios by book-to-market equity and size consecutively.

Figure 1 presents 3-D bar plots of average excess returns, betas, and alphas. Low-BE/ME portfolios underperform relative to high-BE/ME portfolios and big-stocks achieve lower return relative to small-stocks [Figure 1(a)]. If market beta has linear relationship with return, beta should go up consistently from low-BE/ME portfolios to high- BE/ME portfolios and from big-stock portfolios to small-stock portfolios. However, this desirable movement of beta cannot be observed precisely from Figure

1(b) which reflects that beta does not explain cross-section of return well. Moreover, most of the alphas, different from zero, in Figure 1(c) indicate that pricing error in CAPM is not trivial.

Additionally, Figure 2 illustrates the goodness-of-fit of CAPM model with scatter plot of realized average return and predicted return from CAPM for 25 portfolios. As most of the filled circles lie outside the 45° (thick purple) line, we cannot claim that sensitivity to market volatility can fully explain the cross-section of average returns on stock portfolios. As alphas for at least 13 portfolios are different from 0, CAPM does not explain shared variation in average return properly. It suggests that there is existence of additional factors which are relevant for asset pricing (Banz, 1981; Basu, 1983).

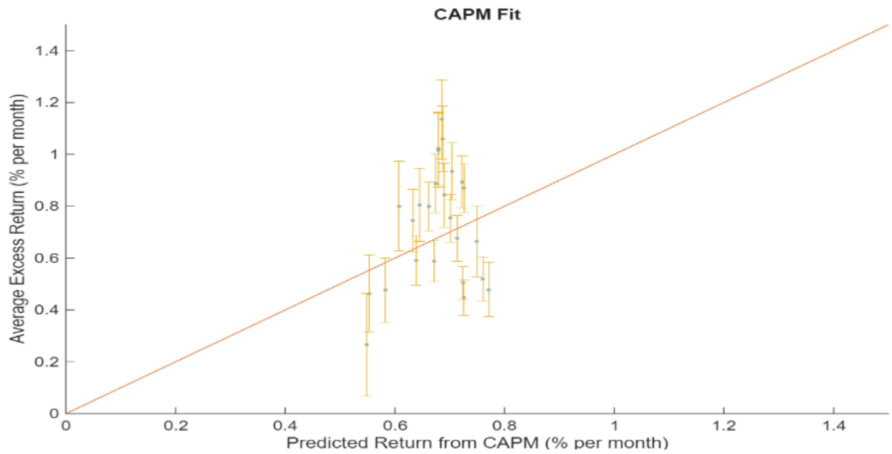

Figure 2. Scatterplot shows the interaction of predicted excess returns (x-axis) and actual average excess returns (y-axis) per month on Fama-French 25 portfolios. High deviation of filled circles from the 45o (thick purple) line indicates low explanatory power of CAPM to explain cross-section of average returns on stock portfolios.

4.2 Fama-French three-factor model

Table 2 presents the coefficients, alphas, and t-statistics from the Fama-French three-factor time-series regression for Fama-French 25 portfolios. Average excess returns tend to be greater for small-size portfolios relative to big-size portfolios, and for high-BE/ME portfolios relative to low-BE/ME portfolios.

Table 2. Summary Statistics and Fama-French Three-Factor Regressions for Simple Monthly Percent Excess Returns on 25 Fama-French Portfolios Formed on Size and BE/ME: July 1963 to November 2013, 605 months.

$$R_{k,t} - Rf = \alpha_k + \beta_k(Rm_t - Rf) + h_k HML_t + s_k SMB_t + \varepsilon_{k,t}$$

	Book-to-market equity *(BE /ME)* quintiles									
Size	Low	2	3	4	High	Low	2	3	4	High
quintiles			$\overline{R(t) - Rf(t)}$					β		
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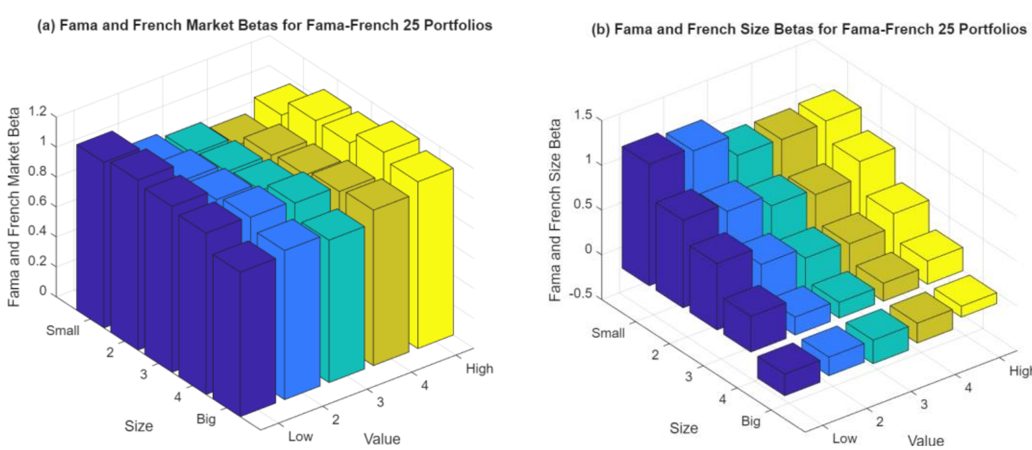
3	0.48	0.80	0.75	0.89	1.06	1.11	1.04	0.99	0.98	1.08
4	0.59	0.59	0.68	0.87	0.84	1.08	1.07	1.05	1.01	1.16
Big	0.45	0.51	0.52	0.48	0.66	0.96	1.00	0.95	1.04	1.11
			s					h		
Small	1.36	1.32	1.08	1.07	1.08	-0.29	0.05	0.30	0.44	0.69
2	0.98	0.89	0.76	0.70	0.88	-0.37	0.12	0.39	0.56	0.80
3	0.74	0.54	0.42	0.39	0.54	-0.42	0.18	0.42	0.60	0.79
4	0.40	0.20	0.18	0.20	0.26	-0.41	0.20	0.44	0.57	0.78
Big	-0.24	-0.21	-0.26	-0.22	-0.12	-0.38	0.10	0.31	0.65	0.81
			α					$t(\alpha)$		
Small	-0.50	-0.02	-0.04	0.15	0.13	-5.21	-0.29	-0.64	2.70	2.28
2	-0.20	-0.02	0.08	0.08	-0.03	-2.85	-0.28	1.34	1.50	-0.53
3	-0.09	0.09	0.00	0.09	0.09	-1.43	1.27	0.05	1.37	1.23
4	0.12	-0.06	-0.05	0.11	-0.09	1.89	-0.85	-0.64	1.57	-1.06
Big	0.17	0.03	0.00	-0.23	-0.17	3.59	0.44	-0.04	-3.44	-1.69

Note: Rm is the value-weighted monthly return (%) of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ. Rf is the one-month Treasury bill rate, observed at the beginning of the month. SMB (small minus big) is the return on the mimicking portfolio for the size factor in stock returns. HML (high minus low) is the return on the mimicking portfolio for the book-to-market factor. Dependent variable: Excess returns on 25 portfolios of stocks formed on size and book-to-market equity. β, h, and s are slopes for market factor, book-to-market equity factor, and size factor, respectively. α is intercept from time-series regression for 25 portfolios. Rolling window for time-series regression is 60 months.

However, this common variation in returns is not captured by market β since the relation between average return and β s for 25 portfolios seems to be flat. Conversely, coefficients on SMB and HML for these portfolios tend to explain variation in stock returns strongly. In every size quintile, the coefficients on HML, which proxies return for the book-to-market equity, increase monotonically from low- to high BE/ME quintiles. Similarly, in every BE/ME quintile, the slopes on SMB, the mimicking return for the size factor, increase monotonically from high negative values for the biggest-size quintiles to high positive values for the smallest-size quintiles. Both HML and SMB clearly capture systematic variation in stock returns, related to size and book-to-market equity, that is neglected by market beta. Moreover, alphas for 25 portfolios in three-factor regression range from -0.50% to 0.17% per month. Only 6 of the 25 alphas differ from 0 by more than 0.14% per month which implies that $Rm - Rf$, SMB, and HML capture shared variation in stock returns more precisely.

Figure 3 displays the coefficients on $Rm - Rf$, HML, and SMB in 3-D plots. Though market beta should be higher for portfolios with higher average return, this relation cannot be observed for all quintiles in Figure 3(a). However, in Figure 3(b), monotonic increase of slopes on BE/ME can be precisely observed from low- to high-BE/ME (x-axis) portfolios. It implies that HML captures systematic variation in stock returns, related to book-to-market equity, which is not captured by market and SMB. Next, Figure 3(c) reveals consistent decrease of slopes on size from small- to big- portfolios (y-axis). SMB tends to capture the variation in stock returns in response to size that is not explained by market and book-to-market equity.



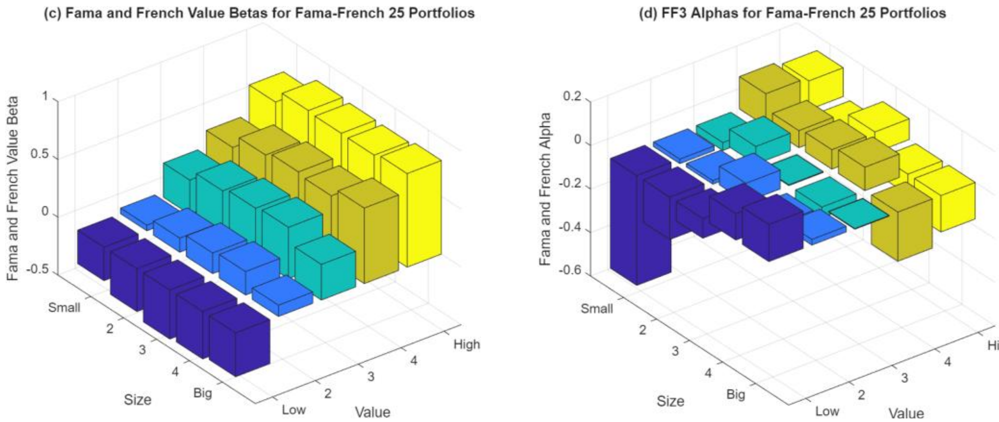


Figure 3. Top panel exhibits market betas for Fama-French 25 portfolios with Fama and French market beta in z-axis. Left part and right part of bottom panel present coefficients on book-to-market equity factor and coefficients on size factor, respectively. All coefficients are generated from time series regression of excess return for each of Fama-French 25 portfolios on excess market return, mimicking returns for the size (SMB) and book-to-market equity (HML) factors. Z-axis in left figure and right figure of bottom panel indicates the Fama and French value (book-to-market equity) beta and Fama and French size beta, respectively. X-axis and Y-axis of all plots show sorting of portfolios by book-to-market equity and size consecutively.

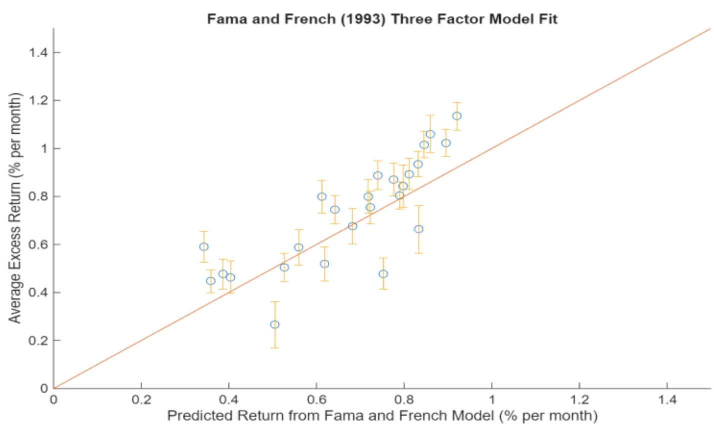


Figure 4. Scatterplot displays the relation between predicted excess returns (x-axis) and actual average excess returns (y-axis) per month on Fama-French 25 portfolios. Error bar on each plot indicates standard errors of respective alphas. Most of the filled circles lie close to the 45o (thick purple) line with relatively low standard errors of alphas which suggests that Fama and French three-factor model has strong explanatory power to explain cross-section of average returns on stock portfolios.

As both HML and SMB capture common variation in average returns of portfolios properly, the predicted returns from regression on the Fama-French three-factor model are more precise which is demonstrated in Figure 4. Predicted returns from Fama-French three-factor model for 25 portfolios are strongly in line with the actual returns. Besides, low standard errors of alphas for most of the portfolios provide evidence that variables in Fama-French three-factor model have significant explanatory power to describe average returns.

4.3 Fama-French five-factor model

Table 3 presents the coefficients, alphas, and t-statistics from the Fama-French five-factor time-series regression for Fama-French 25 portfolios. Average excess returns tend to be greater for small-size portfolios relative to big-size portfolios, and for high-BE/ME portfolios relative to low-BE/ME portfolios with few exceptions.

Table 3. Summary Statistics and Fama-French Five-Factor Regressions for Simple Monthly Percent Excess Returns on 25 Fama-French Portfolios Formed on Size and BE/ME: July 1963 to November 2013, 611 months.

$$R_{k,t} - Rf = \alpha_k + \beta_k(Rm_t - Rf) + h_k HML_t + s_k SMB_t + r_k RMW_t + c_k CMA_t + \varepsilon_{k,t}$$

	Book-to-market equity *(BE /ME)* quintiles									
Size	Low	2	3	4	High	Low	2	3	4	High
quintiles			$\overline{R(t) - Rf(t)}$					β		
Small	0.27	0.80	0.80	1.02	1.13	1.06	0.95	0.94	0.89	0.98
2	0.46	0.74	0.89	0.94	1.02	1.11	1.03	0.99	0.98	1.08
3	0.48	0.80	0.75	0.89	1.06	1.08	1.06	1.01	1.00	1.10
4	0.59	0.59	0.68	0.87	0.84	1.06	1.11	1.09	1.03	1.17
Big	0.45	0.51	0.52	0.48	0.66	0.97	1.03	0.98	1.04	1.06
			s					h		
Small	1.26	1.25	1.09	1.08	1.11	-0.22	0.05	0.27	0.41	0.65
2	0.96	0.93	0.84	0.75	0.91	-0.30	0.11	0.38	0.52	0.81
3	0.72	0.60	0.49	0.45	0.61	-0.31	0.18	0.41	0.55	0.75
4	0.37	0.27	0.24	0.22	0.28	-0.37	0.09	0.37	0.53	0.79
Big	-0.20	-0.16	-0.24	-0.21	-0.18	-0.35	0.00	0.22	0.66	0.96
			r					c		
Small	-0.39	-0.26	0.06	0.05	0.11	-0.17	-0.03	0.05	0.06	0.09
2	-0.07	0.16	0.29	0.20	0.11	-0.17	0.03	0.03	0.09	-0.02
3	-0.09	0.24	0.25	0.23	0.24	-0.26	0.01	0.05	0.12	0.11
4	-0.10	0.27	0.26	0.09	0.09	-0.09	0.27	0.19	0.10	-0.01
Big	0.17	0.20	0.09	0.06	-0.25	-0.06	0.22	0.20	-0.02	-0.35

	α					$t(\alpha)$				
Small	-0.32	0.08	-0.07	0.12	0.07	-3.44	1.16	-1.18	2.10	1.27
2	-0.14	-0.08	-0.03	-0.01	-0.07	-1.94	-1.35	-0.57	-0.24	-1.17
3	-0.01	0.00	-0.10	-0.02	-0.02	-0.11	-0.04	-1.46	-0.32	-0.22
4	0.18	-0.22	-0.18	0.05	-0.12	2.67	-3.00	-2.49	0.76	-1.36
Big	0.12	-0.09	-0.08	-0.24	-0.01	2.59	-1.60	-1.05	-3.58	-0.06

Note: Rm is the value-weighted monthly return (%) of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ. Rf is the one-month Treasury bill rate, observed at the beginning of the month. SMB (small minus big) is the return on the mimicking portfolio for the size factor in stock returns. HML (high minus low) is the return on the mimicking portfolio for the book-to-market factor. RMW (robust minus weak) is the average return on the mimicking portfolio for the operating profitability factor. CMA (conservative minus aggressive) is the average return on the mimicking portfolio for the investment factor. Dependent variable: Excess returns on 25 portfolios of stocks formed on size and book-to-market equity. β, h, s, r, and c are slopes for market factor, book-to-market equity factor, size factor, profitability factor, and investment factor, respectively. α is intercept from time-series regression for 25 portfolios. Rolling window for time-series regression is 60 months.

However, this common variation in returns is not captured by market β presented in Table 3. If we observe beta coefficients for size and value sorted portfolios, we can observe portfolios with higher returns do not have higher market beta and portfolios with lower return do not necessarily have lower market beta since the relation between average return and βs for 25 portfolios seems to be flat. Like Fama-French three factor model, coefficients on SMB and HML factors tend to explain variation in portfolio returns strongly. In every size quintile, the coefficients on HML increase monotonically from low- to high BE/ME quintiles while in every BE/ME quintile, the slopes on SMB increase monotonically from negative values for the largest-size BE/ME quintiles to high positive values for the smallest-size quintiles.

Two additional factors RMW and CMA seem to explain the variation in stock returns. Since, portfolios are sorted based on size and value factor, we cannot read the coefficients for operating profitability (RMW) and investment (CMA) like size (SMB) and value factor (HML). However, if we compare the coefficients for operating profitability (RMW) factor of portfolios with the returns on the portfolios, we can observe portfolios with high operating profitability beta tend to have high returns and portfolios with low operating profitability beta tend to have low returns. However, this patten is not obvious across all the portfolios. Conversely, investment (CMA) factors tend to weakly explain the variation of cross section of portfolio returns meaning that portfolios with higher returns do not necessarily associate with higher investment beta and vice versa. However, it is obvious that five-factor model can clearly capture systematic variation in stock returns by size, value, profitability and investment factors, which is not well captured by market beta. This claim is corroborated by the alphas for 25 portfolios in five-factor regression which range from -0.32% to 0.18% per month. Out of 25 alphas for portfolios, only 7 portfolios have alpha which differ significantly from 0 which implies that $Rm - Rf$, SMB, HML, RMW, and CMA capture common variation in stock returns more precisely.

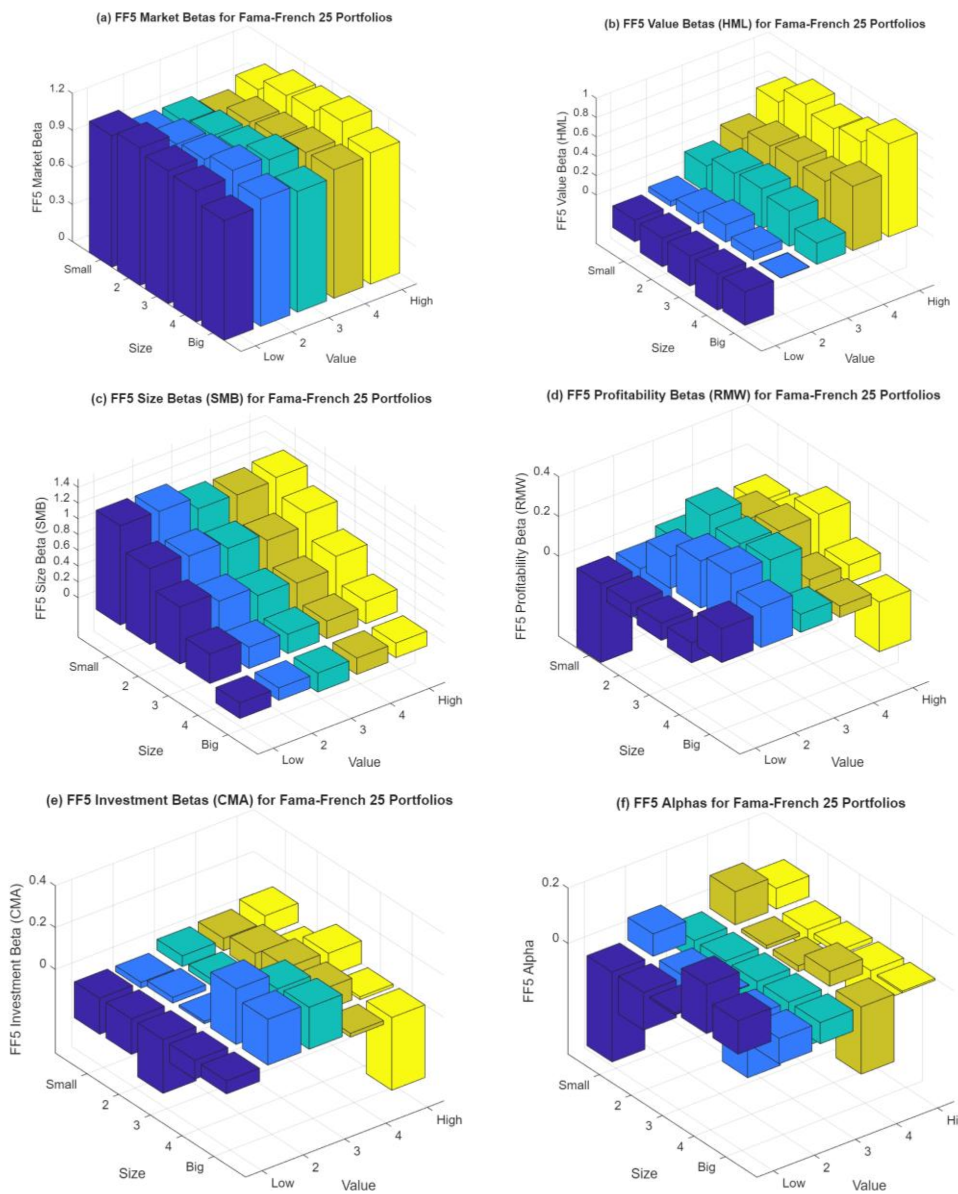


Figure 5. Left part and right part of top panel exhibit market betas for Fama-French 25 portfolios with Fama and French market beta in z-axis, and coefficients on book-to-market equity factor, respectively. Left part and right part of middle panel present coefficients on size factor and coefficients on profitability factor, respectively. Left part and right part of bottom panel present investment betas and alphas, respectively. All coefficients and alphas are generated from time series regression of excess return for each of Fama-French 25 portfolios on excess market return, mimicking returns for the book-to-market equity (HML), size (SMB) factor, profitability factor, and investment factor. Z-axis in first five figures measure coefficients. X-axis and Y-axis of all plots show sorting of portfolios by book-to-market equity and size consecutively.

Figure 5 displays the coefficients on $Rm - Rf$, HML, SMB, RMW, and CMA factors in 3-D plots. It is obvious from Figure 5(a) that market beta cannot capture variation of portfolio returns. However, in Figure 5(b), monotonic pattern of coefficients on BE/ME can be precisely observed from low- to high-BE/ME (x-axis) portfolios. It indicates that HML captures systematic variation in stock returns, related to book-to-market equity, which is not captured by market and SMB. Similarly, Figure 5(c) reveals consistently higher slopes on small sized portfolios relative to large sized portfolios (y-axis). SMB can capture the variation in stock returns in response to size that is not explained by market and book-to-market equity. Figure 5(d) indicates that portfolio beta can apparently capture variation of portfolio returns. Portfolios with higher returns tend to have higher profitability betas and vice versa but this is not obvious for all portfolios across. Similarly, with respect to investment factor, higher portfolio returns are associated with higher coefficients on investment factor and lower portfolio returns are associated with lower coefficients on investment factor. However, for some portfolios this relationship is not robust.

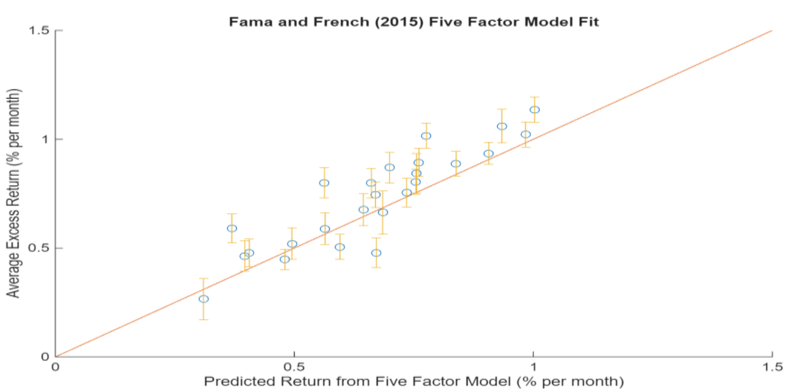


Figure 6. Scatterplot displays the correlation between predicted excess returns (x-axis) and average excess returns (y-axis) per month on Fama-French 25 portfolios. Error bar on each plot indicates standard errors of respective alphas. Most of the circles lie close to the 45o (thick purple) line with relatively low standard errors of alphas which provides evidence that Fama and French five-factor model has strong explanatory power to explain cross-section of average returns on stock portfolios.

As both RMW, and CMA factors in addition to existing HML and SMB factors can capture more common variation in average returns of portfolios, the predicted returns from regression on the Fama-French five-factor model are very close to the actual returns which is demonstrated in Figure 6. Predicted returns from Fama-French five-factor model for 25 portfolios are strongly aligned with the actual returns. Besides, minimum standard errors of alphas for most of the portfolios provide corroborate the hypothesis that Fama-French five-factor model have robust explanatory power to describe cross-sectional variation in average returns.

4.4 Comparison of three models

The CAPM regressions for monthly returns of the 25 Fama-French portfolios on market beta reveal that market beta does not fully account for all the systematic risks to which the portfolios are exposed. The Fama-French three-factor model overcomes this limitation by incorporating size and value factors, providing a more comprehensive explanation of the cross-section of stock returns. Ultimately, the Fama-French five-factor model offers the most thorough explanation of the variation in systematic returns by including market beta, size, value, profitability, and investment factors. Table 4 reports the root mean square alpha and the average R^2 of each of the models for comparison.

Table 4. Comparison of CAPM, Fama-French Three-Factor Model, Five-Factor Model

Model	Root mean square (RMS) alpha	Average R^2
CAPM	0.3200	0.7460
Fama-French three-factor model	0.1419	0.9119
Fama-French five-factor model	0.1256	0.9176

Note: Root mean square alpha and average R2 have been estimated from the time series regression of each model for Fama-French 25 portfolios of stocks formed on size and book-to-market equity. CAPM includes only market factor, Fama-French three-factor model includes market, book-to-market equity, and size factors; July 1963 to November 2013, 611 months; five-factor model includes market, book-to-market equity, size, profitability, and investment factors: July 1963 to November 2013, 605 months. Rolling window for time-series regression is 60 months.

Comparing asset pricing models based on the intercepts in regression is stringent standard (Fama and French, 1993). Fama-French five-factor model has the lowest root mean square (RMS) alpha, which is even less than half of RMS alpha of CAPM. Moreover, three-factor model has lower RMS alpha, also less than half of RMS alpha of CAPM, compared to CAPM. It confirms that Fama-French five-factor model strongly dominates three-factor model and CAPM. However, RMS alpha ignores distribution of residuals which is addressed in Wald or Gibbons-Ross-Shanken test.

R^2 provides evidence on the proportion of common variation in average returns that is explained by factors of model. Average R^2 values of these models are consistent with RMS alpha values as expected. Average R^2 value of 0.9176 for Fama-French five-factor model is the highest among three models. It suggests that this model has the strongest explanatory power relative to three-factor model and CAPM to explain differences in average return on 25 portfolios. It provides strong support for size and value factor in asset pricing models. However, the improvement of explanatory power of Fama-French five-factor model over three-factor model is marginal which shows that inclusion of profitability and investment factor does not capture significantly higher variation of portfolio returns. Nevertheless, R^2 adjusted for degrees of freedom would provide best statistics for comparing models.

The empirical results definitely illustrate the higher explanatory ability of the Fama-French five-factor model relative to both CAPM and a three-factor model integrating size and value characteristics to market factor. The five-factor model has the lowest root mean square alpha, which is less than half that of CAPM, indicating greatly decreased pricing errors. Furthermore, the model gets the greatest average R^2 of 0.9176, explaining nearly 92% of the common variation in average returns across the 25 test portfolios. This large boost in explanatory power emphasizes the importance of including size and book-to-market characteristics beyond the typical market beta in asset pricing models.

The factor premium study gives crucial insights regarding the origins of return volatility in equity markets. Most significantly, our Fama-MacBeth regression results suggest that book-to-market equity emerges as the primary factor in explaining cross-sectional return disparities, with an economically and statistically significant risk premium of 0.4116% each month (t-statistic = 3.20). This finding complements the vast body of work proving the value impact and suggests that book-to-market equity incorporates systematic risk characteristics or behavioral biases that are not accounted for by market beta alone.

4.5 Factor premium

Table 5 reports the slopes of single cross-sectional regression which shows that both book-to-market equity and size have non-zero risk premium for 1963-2013. Market beta does not explain average stock return for same period and the slope for β is negative.

Table 5. Single Cross-Sectional Regression for Simple Monthly Percent Excess Returns on market β, Size, Book-to-Market Equity, Profitability, and Investment: July 1963 to November 2013, 605 months.

$$R_k - Rf = \gamma_0 + \gamma_1\beta_k + \gamma_2 s_k + \gamma_3 h_k + \gamma_4 r_k + \gamma_3 c_k + \eta_k$$

Factor	Coefficient	Standard Error	T-statistic
Intercept (γ_0)	1.0270	0.3956	2.5959
Market Beta (γ_1)	-0.5483	0.3770	-1.4546
Size (γ_2)	0.2338	0.0496	4.7097
Book-to-market Equity (γ_3)	0.3688	0.0601	6.1395
Profitability (γ_4)	0.4627	0.1897	2.4393
Investment (γ_5)	-0.0231	0.2336	-0.0988

Note: β, s, h, r, and s are estimated coefficients on market, size, book-to-market equity, profitability, and investment factors, respectively from the time series regression of Fama-French five-factor model for Fama-French 25 portfolios of stocks formed on size and book-to-market equity. Rolling window for time-series regression is 60 months.

However, standard errors from single cross-sectional regression have drawbacks. Firstly, these tend to be underestimated because cross-sectional correlation and serial correlation in error terms are not corrected. Secondly, the time-varying nature of betas are not recognized. Finally, OLS standard errors are not robust for heteroskedasticity (Petersen, 2009). Table 6 presents the revised estimates of cross-sectional regression using Fama-MacBeth approach, which addresses the drawbacks identified above.

Table 6. Month-by-Month Cross-Sectional Regressions for Simple Monthly Percent Excess Returns on market β, Book-to-Market Equity, and Size: July 1963 to November 2013, 605 months.

$$R_k - Rf = \gamma_0 + \gamma_1\beta_k + \gamma_2 s_k + \gamma_3 h_k + \gamma_4 r_k + \gamma_3 c_k + \eta_k$$

Factor	Coefficient	Standard Error	T-statistic
Intercept (γ_0)	0.5848	0.3207	1.8234
Market Beta (γ_1)	-0.0770	0.3681	-0.2090
Size (γ_2)	0.1208	0.1334	0.9058
Book-to-market Equity (γ_3)	0.3881	0.1276	3.0408
Profitability (γ_4)	0.4970	0.1499	3.3163
Investment (γ_5)	0.3610	0.1803	2.0018

β, s, h, r, and s are estimated coefficients on market, size, book-to-market equity, profitability, and investment factors, respectively from the time series regression of Fama-French five-factor model for Fama-French 25 portfolios of stocks formed on size and book-to-market equity. Rolling window for time-series regression is 60 months. The slope parameter estimates (the lambdas) from month-by-month cross-sectional regression is the time-series average of the monthly regression coefficients for July 1963 to November 2013, and the t-statistic is the average slope divided by its time-series standard error. Standard error is the standard deviation over time divided by the square root of the number of time-series estimates of the lambdas which is based on the Fama and MacBeth (1973) technique.

The estimated slopes are different in Fama-MacBeth regression compared to OLS regression. Although all slopes have similar signs in both regressions, explanatory power of size factor decreases in Fama-MacBeth regression for lower slope and higher standard error. This finding does not support Fama and French (1992). Moreover, slope of market beta is negative, and it does not explain average stock returns. In contrast, the average risk premium on book-to-market equity alone is 0.3881% with t-statistic of 3.04. This is because high BE/ME stocks typically have persistent poor earnings which is reversed later (Fama and French, 1995). Conversely, Lakonishok et al. (1994) attribute this risk premium to mispricing. However, Jegadeesh *et al.*, (2019) note that risk premium estimates from Fama-Macbeth regression are not unbiased.

The consistently negative slope coefficient for market beta throughout our parameters violates the fundamental prediction of CAPM that more systematic risk should be compensated with higher expected returns. This conclusion corresponds with the broader literature showing the "flat" security market line and shows that market beta alone provides insufficient information for interpreting cross-sectional return variability in our sample period.

Interestingly, results regarding the size factor show a more nuanced picture than previously described by Fama and French (1992). While the size component exhibits explanatory power in single cross-sectional regression, its relevance declines in the more robust Fama-MacBeth framework due to increased standard errors and lower slope coefficients. This attenuation shows that the size effect may be less robust than the book-to-market effect when accounting for cross-sectional correlation and time-varying factor loadings, underscoring the significance of adopting rigorous econometric approaches in asset pricing assessments. Finally, significance of profitability and investment factor reinforces support for the evidence documented by Fama and French (2015).

5.0 Conclusion

This paper presents thorough empirical evidence on the relative performance of the Capital Asset Pricing Model (CAPM), the Fama-French Three-Factor model, and the Fama-French Five-Factor model in explaining the cross-section of stock returns for the period from July 1963 to November 2013. Using the Fama-MacBeth methodology to evaluate these models on 25 portfolios based on size and book-to-market equity characteristics, this study gives some key insights that contribute to the continuing discourse in asset pricing literature.

The results provide evidence that there are significant limitations of the CAPM in explaining the cross-section of expected returns, with substantial alpha estimates across portfolios indicating systematic pricing errors. In conclusion, our study reinforces the substantial body of evidence supporting the Fama-French three-factor model as a significant improvement over CAPM in explaining equity return patterns. The significance of book-to-market equity, profitability, and investment in driving cross-sectional return variations, combined with the low explanatory power of market beta and size underlines the complexity of risk-return linkages in financial markets. From a practical standpoint, these findings have major implications for portfolio management, performance evaluation, and risk assessment. The superior performance of the five-factor model suggests that investment professionals should incorporate value, profitability, and investment factors in their risk models and performance attribution frameworks. For corporate finance applications, we suggest the use of multifactor models in cost of capital calculation, particularly underlining the importance of book-to-market features in determining necessary returns.

However, certain limitations deserve considered in interpreting our results. First, the Fama-MacBeth methodology, while addressing major econometric difficulties, may generate its own biases in risk premium estimation. Second, this analysis focuses on a specific set of portfolios and time period, and the generalizability of findings to other asset classes, international markets, or alternative time periods requires more investigation. Third, the theoretical interpretation of the components—whether they represent systematic risk factors or behavioral anomalies—remains an open subject that our empirical study cannot definitively address.

Future research could extend this analysis in various directions. First, including additional characteristics such as momentum and psychological factors could provide insights into whether the explanatory power of the five-factor model can be further increased. Second, investigating the time-varying nature of factor premiums and their relationship to macroeconomic conditions could offer deeper insights into the economic causes of these return patterns. Third, employing different methodological approaches, such as GMM estimates or Bayesian techniques, could provide robustness checks and potentially overcome some drawbacks of the Fama-MacBeth approach.

These findings contribute to our understanding of asset pricing and provide valuable guidance for both academic researchers and practitioners seeking to better understand and predict equity returns. As financial markets continue to evolve, the ongoing refinement and testing of asset pricing models remains vital for increasing both theoretical understanding and practical applications in finance.

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